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A RUBBER CONE UNDER THE TENSION OF A CONCENTRATED FORCE

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Abstract—Using the constitutive equation given by Gao [*Theor. Appl. Fracture Mech.* **14**, 219–231 (1990)] for rubber-like materials, this paper analyses the stress distribution and the deformation pattern near the apex of a cone that is under the tension of a concentrated force. The problem is treated as an axial symmetry case. Infinite strain is taken into account and the analytical result is presented. When the cone angle is 180° the solution of the nonlinear Boussinesq problem is obtained.

1. INTRODUCTION

A body acted on by a concentrated force is a very important subject in the linear theory of elasticity. Michell (1902) solved a typical problem for the two-dimensional case, i.e. a wedge under the action of a concentrated force. Boussinesq (1885) gave a very famous solution to the three-dimensional case, i.e. a half space under the action of a concentrated force. All of the solutions to the problems with concentrated load possess singularity. Therefore, these solutions become invalid when the singular points are approached. However, on the other hand, the singular solution can still reflect the essential tendency of the stress and strain near the singular points. For ordinary engineering materials, the linear theory of elasticity is sufficient because the failure always occurs at a strain that is not very large. For rubber-like materials which can endure large deformation, the nonlinear geometry must be taken into account, especially for problems with concentrated forces, where the infinite strain governs the behavior of the singular point and the problem becomes very complicated. There are three obstacles to the solution of the concentrated force problem for rubber-like materials. First, when strain tends to infinity the ordinary constitutive relation may lose its meaning. Second, the inherently geometric nonlinearity must be described by some proper mathematical formulae. Finally, the deformation pattern near the singular point is difficult to construct. Gao (1990) gave a constitutive relation for rubber-like materials that is valid for infinite deformation, where the stress is decomposed into a hydrostatic part and a deviatoric part. Using this constitutive relation, Gao (1990) analysed the crack tip field of rubber-like materials for the plane strain case. Using the same constitutive equation, Gao and Durban (1994) analysed the crack tip field in a rubber sheet (plane stress case), and the solution was presented in closed mathematical form. The concentrated force problem of rubber-like materials for the two-dimensional case was investigated by Gao (1994), who treated a rubber wedge under the tension of a concentrated load at its tip. A completely new deformation pattern was obtained by Gao (1994). Further, the compression case was solved by Gao and Gao (1994), i.e. a rubber wedge under the action of a concentrated load that is directed inside the material domain. It must be pointed out that the compression problem is quite different from the tension case in deformation pattern; the former consists of an expansion sector and two narrowing sectors but the latter consists only of a narrowing sector. In the present paper a rubber cone under the tension of a concentrated load at its apex will be considered. The constitutive relation given by Gao (1990) is used to obtain the mathematical solution. The problem of a cone under tension is similar to that of a wedge under tension, i.e. the apex field consists of only a narrowing domain.

2. FORMULATION

Under consideration is a three-dimensional domain of material. $\mathbf{P}$ and $\mathbf{p}$ denote the position vectors of a material point before and after deformation, respectively. $X^i(i = 1, 2, 3)$ denotes the Lagrangian coordinate. Then we can introduce two local triads:

$$\mathbf{P}_i = \frac{\partial \mathbf{P}}{\partial X^i}, \quad \mathbf{p}_i = \frac{\partial \mathbf{p}}{\partial X^i}. \tag{1}$$

The displacement gradient is defined as

$$\mathbf{F} = \mathbf{p}_i \otimes \mathbf{P}^i, \tag{2}$$

where the summation rule is implied, $\mathbf{P}^i$ is the conjugate base of $\mathbf{P}_i$ and $\otimes$ is the dyadic symbol. Further, the right and left Cauchy–Green strain tensors are denoted by $\mathbf{D}$ and $\mathbf{d}$:

$$\mathbf{D} = \mathbf{F}^{\mathrm{T}} \cdot \mathbf{F}, \quad \mathbf{d} = \mathbf{F} \cdot \mathbf{F}^{\mathrm{T}}, \tag{3}$$

in which the superscript T indicates transposition. It can be proved that $\mathbf{D}$ and $\mathbf{d}$ possess the same invariants:

$$I_j = \mathbf{D}^j : \mathbf{E} = \mathbf{d}^j : \mathbf{E}, \quad j = 1, 2, 3, \tag{4}$$

where the colon denotes dual scale product and $\mathbf{E}$ is a unit tensor.

A new form of strain energy per unit undeformed volume is given by Gao (1990)

$$W = A[(I/K^{1/3})^n - 3^n] + B(K-1)^m K^{-q}, \tag{5}$$

in which

$$I = I_1, \quad K = \tfrac{1}{6}(I_1^3 - 3I_1I_2 + 2I_3) \tag{6}$$

and A, B, n, m and q are non-negative material constants.

The Kirchhoff and Cauchy stresses σ and τ can be obtained from the strain energy W:

$$\sigma = 2\frac{\partial W}{\partial \mathbf{D}}, \quad \tau = K^{-1/2}\mathbf{F} \cdot \sigma \cdot \mathbf{F}^{\mathrm{T}}. \tag{7}$$

Using eqns (5)–(7) we obtain

$$\tau = 2nAI^{n-1}K^{-(n/3)-1/2}\left(\mathbf{d} - \frac{I}{3}\mathbf{E}\right) + 2B(K-1)^{m-1}K^{-q-(1/2)}[(m-q)K+q]\mathbf{E}. \tag{8}$$

Evidently, in eqn (8), the stress tensor is decomposed into a deviatoric part and spherical part. For the case of small deformation, when $m = 2$, eqn (8) becomes the linear elastic relation, and we have the Young's modulus and Poisson's ratio:

$$E = \frac{72n3^nAB}{36B + n3^nA}, \quad v = \frac{18B - n3^nA}{36B + n3^nA}. \tag{9}$$

From eqns (9) we can see $18B \geqslant n3^nA$ is required to ensure $v \geqslant 0$. The equilibrium equation is

$$\left(\mathbf{p}^i \frac{\partial}{\partial X^i}\right) \cdot \tau = 0. \tag{10}$$

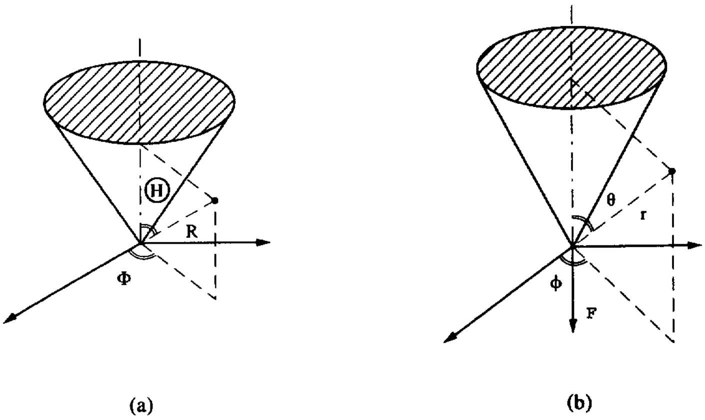

Fig. 1. A revolutionary cone. (a) Before loading. (b) After loading.

3. DEFORMATION PATTERN

Figure 1a shows a rubber cone without load, and Fig. 1b is the cone under tension of a concentrated force $\mathscr{F}$. We assume that the cone possesses revolutionary shape and the deformation is axial symmetric. Let Θ_0 denote the half angle of the cone apex before deformation. Both (R, Θ, Φ) and (r, θ, φ) are taken as the Lagrangian coordinates, such that (R, Θ, Φ) are spherical coordinates before deformation while (r, θ, φ) are spherical coordinates after deformation (see Fig. 1). We presume that after deformation the cone becomes very sharp so that the deformation can be described by the following mapping functions:

$$\begin{aligned} R &= r^{1+\beta} f(\zeta) \\ \Theta &= g(\zeta) \\ \Phi &= \varphi, \quad \zeta = \theta / r^{\alpha}, \end{aligned} \tag{11}$$

in which α, β are constants to be determined, f and g are unknown functions. Let

$$\mathbf{P}_R = \frac{\partial \mathbf{P}}{\partial R}, \quad \mathbf{P}_\Theta = \frac{\partial \mathbf{P}}{\partial \Theta}, \quad \mathbf{P}_\Phi = \frac{\partial \mathbf{P}}{\partial \Phi} \tag{12}$$

$$\mathbf{P}_r = \frac{\partial \mathbf{P}}{\partial r}, \quad \mathbf{P}_\theta = \frac{\partial \mathbf{P}}{\partial \theta}, \quad \mathbf{P}_\varphi = \frac{\partial \mathbf{P}}{\partial \varphi} \tag{13}$$

$$\mathbf{p}_r = \frac{\partial \mathbf{p}}{\partial r}, \quad \mathbf{p}_\theta = \frac{\partial \mathbf{p}}{\partial \theta}, \quad \mathbf{p}_\varphi = \frac{\partial \mathbf{p}}{\partial \varphi} \tag{14}$$

$$\mathbf{e}_R = \mathbf{P}_R, \quad \mathbf{e}_\Theta = \frac{1}{R}\mathbf{P}_\Theta, \quad \mathbf{e}_\Phi = \frac{1}{R\sin\Theta}\mathbf{P}_\Phi \tag{15}$$

$$\mathbf{e}_r = \mathbf{p}_r, \quad \mathbf{e}_\theta = \frac{1}{r}\mathbf{p}_\theta, \quad \mathbf{e}_\varphi = \frac{1}{r\sin\theta}\mathbf{p}_\varphi, \tag{16}$$

then from eqns (13), (11), (12) and (15) we can obtain

$$\begin{aligned} \mathbf{P}_r &= r^{\beta}\{\mathbf{e}_R[(1+\beta)f - \alpha\zeta f'] - \mathbf{e}_\Theta \alpha\zeta f g'\} \\ \mathbf{P}_\theta &= r^{1+\beta-\alpha}(\mathbf{e}_R f' + \mathbf{e}_\Theta f g') \\ \mathbf{P}_\varphi &= r^{1+\beta} f \sin g \, \mathbf{e}_\Phi. \end{aligned} \tag{17}$$

From eqn (17) we obtain the contravariant bases,

$$\mathbf{P}^r = V^{-1} r^{-\beta} (\mathbf{e}_R f g' - \mathbf{e}_\Theta f')$$

$$\mathbf{P}^\theta = V^{-1} r^{\alpha-\beta-1} \{\mathbf{e}_R \alpha\zeta f g' + \mathbf{e}_\Theta [(1+\beta)f - \alpha\zeta f']\}$$

$$\mathbf{P}^\varphi = \frac{r^{-1-\beta}}{f \sin g} \mathbf{e}_\Phi, \tag{18}$$

in which

$$V = (1+\beta) f^2 g'. \tag{19}$$

Using eqns (2), (3), (14), (16), (11) and (18), the strain is calculated:

$$\begin{aligned}\mathbf{d} = {} & V^{-2} r^{-2\beta} U \mathbf{e}_r \otimes \mathbf{e}_r + V^{-2} r^{2\alpha-2\beta} \{(\alpha\zeta f g')^2 \\ & + [(1+\beta) f - \alpha\zeta f']^2\} \mathbf{e}_\theta \otimes \mathbf{e}_\theta + V^{-2} r^{\alpha-2\beta} \{\alpha\zeta f^2 g'^2 \\ & - f'[(1+\beta) f - \alpha\zeta f']\} (\mathbf{e}_r \otimes \mathbf{e}_\theta + \mathbf{e}_\theta \otimes \mathbf{e}_r) \\ & + r^{2\alpha-2\beta} \frac{\zeta^2}{f^2 \sin^2 g} \mathbf{e}_\varphi \otimes \mathbf{e}_\varphi,\end{aligned} \tag{20}$$

in which

$$U = f'^2 + f^2 g'^2. \tag{21}$$

From eqns (6), (4) and (20), we have

$$I = r^{-2\beta} V^{-2} U, \quad K = r^{4\alpha-6\beta} V^{-2} Q^2, \tag{22}$$

where

$$Q = \frac{\zeta}{f \sin g}. \tag{23}$$

Further, we assume that $K \to +\infty$ when $r \to 0$, then eqn (8) is reduced to

$$\tau = 2nAI^{n-1} K^{-(n/3)-(1/2)} \left(\mathbf{d} - \frac{I}{3}\mathbf{E}\right) + 2BsK^{s-(1/2)}\mathbf{E}, \tag{24}$$

where

$$s = m - q. \tag{25}$$

From eqns (25) and (22) we have the following estimation:

$$\tau \sim (\ldots) r^{-2n\beta-((n/3)+(1/2))(4\alpha-6\beta)} + (xxx) r^{(s-(1/2))(4\alpha-6\beta)}. \tag{26}$$

It is required that the two terms in eqn (26) possess the same order when $r \to 0$; therefore,

$$\alpha = \frac{9s\beta}{2(3s+n)}. \tag{27}$$

On the other hand, the resultant force acting at the cone apex must have a finite value, so

$$\tau^{rr} \sim r^{-2(1-\alpha)}. \tag{28}$$

From eqns (26), (27) and (28), it follows that

$$\alpha = \frac{3s}{n(2s-1)-3s}, \quad \beta = \frac{2}{3} \cdot \frac{3s+n}{n(2s-1)-3s}. \tag{29}$$

Taking the dominant terms only, eqn (24) can be written as

$$\tau = \mathbf{r}^{-\lambda}[T^{rr}\mathbf{e}_r \otimes \mathbf{e}_r + T^{\theta\theta}\mathbf{e}_\theta \otimes \mathbf{e}_\theta + r^\alpha T^{r\theta}(\mathbf{e}_r \otimes \mathbf{e}_\theta + \mathbf{e}_\theta \otimes \mathbf{e}_r) + T^{\varphi\varphi}\mathbf{e}_\varphi \otimes \mathbf{e}_\varphi], \tag{30}$$

where

$$\lambda = \frac{2n(2s-1)}{n(2s-1)-3s} \tag{31}$$

$$T^{rr} = \frac{4nA}{3}(V^{-2}U)^n(V/Q)^{\frac{2n}{3}+1} + 2Bs(V/Q)^{1-2s}$$

$$T^{\theta\theta} = T^{\varphi\varphi} = -\frac{2nA}{3}(V^{-2}U)^n(V/Q)^{\frac{2n}{3}-1} + 2Bs(V/Q)^{1-2s}$$

$$T^{r\theta} = 2nAV^{-2n}U^{n-1}(V/Q)^{\frac{2n}{3}+1}\{\alpha\zeta f^2 g'^2 - f'[(1+\beta)f - \alpha\zeta f']\}. \tag{32}$$

The equilibrium equation is

$$\left(\mathbf{e}_r \frac{\partial}{\partial r} + \frac{1}{r}\mathbf{e}_\theta \frac{\partial}{\partial \theta} + \frac{1}{r\sin\theta}\mathbf{e}_\varphi \frac{\partial}{\partial \varphi}\right) \cdot \tau = 0. \tag{33}$$

4. COORDINATE TRANSFORMATION

In order to reveal the essence of the deformation, we introduce a new coordinate system (η, ζ, φ) such that one family of coordinate surfaces is

$$\zeta = \theta/r^\alpha = \text{const.} \tag{34}$$

Another family of coordinate surfaces is

$$\varphi = \text{const.} \tag{35}$$

The third family of coordinate surfaces is taken to be perpendicular to the above families. According to this condition we can obtain the following formula:

$$\eta = r\left(1 + \frac{\alpha}{2}\theta^2 + \frac{\alpha^2}{8}\theta^4 + \frac{\alpha^3}{48}\theta^6 + \dots\right)$$

$$\zeta = \theta/r^\alpha. \tag{36}$$

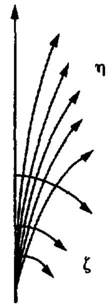

Fig. 2. η, ζ coordinates.

On the cross-section $\varphi = \text{const}$, the coordinate lines of η and ζ are shown in Fig. 2. Since only the vicinity of $\theta = 0$ is considered, we can use the following approximately inverse expression:

$$r = \eta\left[1 - \frac{\alpha}{2}(\zeta\eta^{\alpha})^2\right]$$

$$\theta = \zeta\eta^{\alpha}. \tag{37}$$

Using eqn (37), the base vectors can be calculated:

$$\mathbf{p}_\eta = \mathbf{e}_r + \alpha\theta\mathbf{e}_\theta, \quad \mathbf{p}_\zeta = \eta^{(1+\alpha)}(-\alpha\theta\mathbf{e}_r + \mathbf{e}_\theta), \tag{38}$$

then

$$\mathbf{p}^\eta = \mathbf{p}_\eta, \quad \mathbf{p}^\zeta = \eta^{-2-2\alpha}\mathbf{p}_\zeta \tag{39}$$

$$\mathbf{e}_\eta = \mathbf{e}_r + \alpha\theta\mathbf{e}_\theta, \quad \mathbf{e}_\zeta = -\alpha\theta\mathbf{e}_r + \mathbf{e}_\theta \tag{40}$$

$$\mathbf{e}_r = \mathbf{e}_\eta - \alpha\zeta\eta^{\alpha}\mathbf{e}_\zeta, \quad \mathbf{e}_\theta = \alpha\zeta\eta^{\alpha}\mathbf{e}_\eta + \mathbf{e}_\zeta. \tag{41}$$

Substituting eqns (41) and (37) into eqn (20) we obtain

$$\mathbf{d} = \eta^{-2\beta}V^{-2}[U\mathbf{e}_\eta \otimes \mathbf{e}_\eta - \eta^{\alpha}(1+\beta)ff'(\mathbf{e}_\eta \otimes \mathbf{e}_\zeta + \mathbf{e}_\zeta \otimes \mathbf{e}_\eta) + \eta^{2\alpha}(1+\beta)^2 f^2 \mathbf{e}_\zeta \otimes \mathbf{e}_\zeta] + \eta^{2\alpha-2\beta}Q^2\mathbf{e}_\varphi \otimes \mathbf{e}_\varphi. \tag{42}$$

From eqns (42) and (24) the stress is rewritten as

$$\tau = \eta^{-\lambda}[T^{\eta\eta}\mathbf{e}_\eta \otimes \mathbf{e}_\eta + T^{\zeta\zeta}\mathbf{e}_\zeta \otimes \mathbf{e}_\zeta + T^{\varphi\varphi}\mathbf{e}_\varphi \otimes \mathbf{e}_\varphi + \eta^{\alpha}T^{\zeta\eta}(\mathbf{e}_\zeta \otimes \mathbf{e}_\eta + \mathbf{e}_\eta \otimes \mathbf{e}_\zeta)], \tag{43}$$

in which

$$T^{\eta\eta} = T^{rr}, \quad T^{\zeta\zeta} = T^{\theta\theta} \quad T^{\zeta\eta} = -2nAV^{-2n}U^{n-1}(V/Q)^{\frac{2n}{3}+1}(1+\beta)ff'. \tag{44}$$

5. EQUILIBRIUM EQUATION AND SOLUTION

In the (η, ζ, φ) system, the equilibrium equation can be written as

$$\left(\mathbf{p}^\eta\frac{\partial}{\partial\eta} + \mathbf{p}^\zeta\frac{\partial}{\partial\zeta} + \mathbf{p}^\varphi\frac{\partial}{\partial\varphi}\right)\cdot\tau = 0 \tag{45}$$

or

$$\left(\eta^{-1-\alpha}\mathbf{e}_\zeta\frac{\partial}{\partial\zeta}+\mathbf{e}_\eta\frac{\partial}{\partial\eta}+\eta^{-1-\alpha}\frac{1}{\zeta}\mathbf{e}_\varphi\frac{\partial}{\partial\varphi}\right)\cdot\tau = 0. \tag{46}$$

In order to obtain the final form of eqn (45), we need the following relations:

$$\begin{aligned}
&\frac{\partial\mathbf{e}_\eta}{\partial\eta} = \alpha(1+\alpha)\zeta\eta^{\alpha-1}\mathbf{e}_\zeta, \quad \frac{\partial\mathbf{e}_\eta}{\partial\zeta} = (1+\alpha)\eta^2\mathbf{e}_\zeta, \quad \frac{\partial\mathbf{e}_\varphi}{\partial\eta} = 0,\\
&\frac{\partial\mathbf{e}_\zeta}{\partial\eta} = -\alpha(1+\alpha)\zeta\eta^{\alpha-1}\mathbf{e}_\eta, \quad \frac{\partial\mathbf{e}_\zeta}{\partial\zeta} = -(1+\alpha)\eta^2\mathbf{e}_\eta, \quad \frac{\partial\mathbf{e}_\varphi}{\partial\zeta} = 0,\\
&\frac{\partial\mathbf{e}_\eta}{\partial\varphi} = (1+\alpha)\zeta\eta^2\mathbf{e}_\varphi, \quad \frac{\partial\mathbf{e}_\zeta}{\partial\varphi} = \mathbf{e}_\varphi, \quad \frac{\partial\mathbf{e}_\varphi}{\partial\varphi} = -\mathbf{e}_\zeta-(1+\alpha)\zeta\eta^\alpha\mathbf{e}_\eta.
\end{aligned} \tag{47}$$

Substituting eqns (43) and (47) into (46) it follows that

$$\begin{aligned}
&\frac{\mathrm{d}T^{\zeta\eta}}{\mathrm{d}\zeta}+\frac{1}{\zeta}T^{\zeta\eta}-(1+\alpha)(T^{\zeta\zeta}+T^{\varphi\varphi}) = 0\\
&\frac{\mathrm{d}T^{\zeta\zeta}}{\mathrm{d}\zeta}+\frac{1}{\zeta}(T^{\zeta\zeta}-T^{\varphi\varphi}) = 0.
\end{aligned} \tag{48}$$

Using eqns (44) and (32), the second of eqns (48) becomes

$$\frac{\mathrm{d}T^{\zeta\zeta}}{\mathrm{d}\zeta} = 0; \tag{49}$$

therefore, eqns (48) can be written as

$$\begin{aligned}
&T^{\zeta\zeta} = T^{\varphi\varphi} = \text{const}\\
&\frac{\mathrm{d}T^{\zeta\eta}}{\mathrm{d}\zeta}+\frac{1}{\zeta}T^{\zeta\eta}-2(1+\alpha)T^{\zeta\zeta} = 0.
\end{aligned} \tag{50}$$

The boundary conditions of the problem are

$$T^{\zeta\zeta} = T^{\zeta\eta} = 0 \quad \text{at} \quad \Theta = \Theta_0\,(g(\zeta) = \Theta_0). \tag{51}$$

From eqns (50) and (51) we obtain

$$T^{\zeta\eta} \equiv 0, \quad T^{\zeta\zeta} \equiv 0. \tag{52}$$

Noting eqns (44), (32) and (52), it follows that

$$\begin{aligned}
&f = f_0 = \text{const}\\
&\frac{nA}{3}U^n - sBV^{\frac{4n}{3}-2s}Q^{\frac{2n}{3}+2s} = 0.
\end{aligned} \tag{53}$$

The first of eqns (53) implies that

$$U = f_0^2 g'^2, \quad V = (1+\beta) f_0^2 g', \quad Q = \frac{\zeta}{f_0 \sin g}. \tag{54}$$

From eqn (54) and the second of eqns (53), we obtain

$$\frac{1}{\zeta} g' \sin g = H = \text{const}, \tag{55}$$

where

$$H = (1+\beta)^{2(1-(\alpha/\beta))} \left(\frac{3sB}{nA}\right)^{\frac{3}{2(3s+n)}} f_0^{-\frac{2\alpha}{\beta}}. \tag{56}$$

Substituting eqns (54)–(56) into eqn (44) we have

$$T^{rr} = T^{\eta\eta} = 2nA \left(\frac{3sB}{nA}\right)^{\frac{2n+3}{2(3s+n)}} [(1+\beta) f_0]^{\frac{-\lambda}{\beta}}. \tag{57}$$

Now we calculate the resultant force $\mathscr{F}$ acting at the apex of the cone:

$$\mathscr{F} = 2\pi \int (\tau^{rr} \cos\theta - \tau^{r\theta} \sin\theta) r^2 \theta \, d\theta \doteq 2\pi T^{\eta\eta} \int \zeta \, d\zeta. \tag{58}$$

Further, using eqn (55), we can write eqn (58) as

$$\mathscr{F} = \frac{2\pi}{H} T^{\eta\eta} \int_0^{\Theta_0} \sin\Theta \, d\Theta = \frac{2\pi}{H} T^{\eta\eta} (1 - \cos\Theta_0). \tag{59}$$

Substituting eqns (56) and (57) into eqn (59), it follows that

$$f_0 = \left[\frac{\mathscr{F}}{4n\pi A (1-\cos\Theta_0)}\right]^{-\frac{\beta}{2}} \left(\frac{3sB}{nA}\right)^{\frac{n\beta}{2(3s+n)}} (1+\beta)^{-1-\beta}. \tag{60}$$

Using eqn (55), we obtain

$$\cos g(\zeta) = 1 - \frac{\zeta^2}{2} \cdot \left(\frac{\mathscr{F}}{4n\pi A (1-\cos\Theta_0)}\right)^{\alpha} \left(\frac{3sB}{nA}\right)^{-\frac{\alpha}{2s}} (1+\beta)^{2(1+\alpha)}. \tag{61}$$

The maximum value of ζ is

$$\zeta_0 = \left[\frac{2}{H}(1-\cos\Theta_0)\right]^{1/2}. \tag{62}$$

6. DISCUSSION AND CONCLUSION

Under tension of a concentrated load, the apex of a cone of rubber-like material will become very sharp. The stress state near the cone apex is unidirectional tension, i.e. $\tau^{rr} \sim r^{-2(1+\alpha)}$, $\tau^{r\theta} \sim r^{\theta\theta} \sim 0$. This stress state is similar to that for the small deformation case, i.e. $\tau^{rr} \sim r^{-1}$, $\tau^{r\theta} = \tau^{\theta\theta} = 0$. A significant difference between rubber-like materials (large

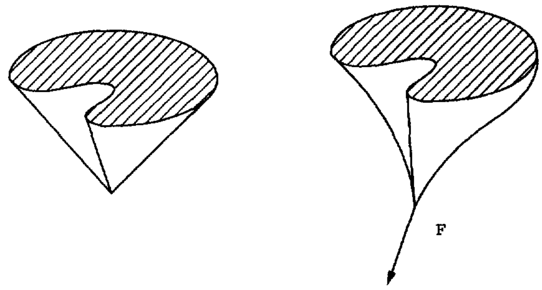

Fig. 3. An arbitrary cone. (a) Before loading. (b) After loading.

strain) and ordinary materials (small strain) is that the former must suffer tremendous deformation to bear the load.

The analysis given in this paper is only for the case of a revolutionary cone under the tension of a force along its axis. However, we can presume that the result obtained here ($f = f_0$ = const) is also valid for the general case, i.e. a cone with arbitrary shape under the tension of a force, as shown in Fig. 3. For the general case we can introduce the following mapping functions:

$$R = r^{1+\beta} f(\zeta, \varphi)$$
$$\Theta = g(\zeta, \varphi), \quad \zeta = \theta / r^{\alpha}$$
$$\Phi = h(\zeta, \varphi). \tag{63}$$

The strict analysis of the general case will be cumbersome. We can expect that the equilibrium equations and the boundary conditions on the cone surface will be satisfied by

$$\frac{\partial f}{\partial \zeta} = 0, \quad \frac{\partial f}{\partial \varphi} = 0; \tag{64}$$

i.e. we still have $f = f_0$ = const, therefore the stress state is also uniaxial tension.

If the concentrated load is directed inside of the cone (compression) the deformation pattern will be quite different from that obtained in this paper. The analysis of the compression problem will be given in another paper by Liu and Gao (1994).

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